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Direct Numerical Simulation and Modeling of a Nonequilibrium Turbulent Plasma

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We investigate the possibility of modeling a low-pressure nonequilibrium turbulent plasma using a probability density function (pdf). Direct numerical simulation (DNS) is used to understand and to model these reactive flows. The pdf transport equation needs a closure for micromixing to capture diffusive and conductive mechanisms occurring at the small scales of the turbulence. A linear relaxation model is retained for micromixing in the case of a recombining plasma in agreement with the results from DNS.

Nomenclature

A = neutral species
A^+ = ion
Da = Damköhler characterizing recombination
Da_{relax} = Damköhler characterizing thermal relaxation
d_{ion} = ionization degree of reference, $d_{\text{ion}} = \rho^{\star}_{A^+,\text{ref}}/\rho^{\star}_{\text{ref}} = Y^{\star}_{A^+,\text{ref}}$
E = total energy, $E = E^{\star}/(\rho^{\star}_{\text{ref}} U^{\star 2}_{\text{ref}}) = \rho ei + 0.5\rho U_i U_i$
e_e = internal energy of electrons, $e_e = T_e/\gamma$
ei = internal energy, $ei = ei^{\star}/U^{\star 2}_{\text{ref}} = (Y_A + d_{\text{ion}} Y_{A^+})T/\gamma + d_{\text{ion}} Y_e T_e/\gamma)$
e_{rec} = recombination energy released in electron gas
$e^0_{A^+}$ = enthalpy of formation of ions
h_α = enthalpy of species α
k = kinetic energy of turbulence
$\langle P \rangle$ = probability density function (Favre averaging)
$\tilde{P}$ = probability density function (Reynolds averaging)
Pr = Prandtl
p = pressure, $p^{\star}/\rho^{\star}_{\text{ref}} U^{\star 2}_{\text{ref}}$
Q_{exc} = thermal energy exchange source term
Re = acoustic Reynolds number, $U^{\star}_{\text{ref}} \rho^{\star}_{\text{ref}} L_{\text{ref}}/\mu^{\star}_{\text{ref}}$
r_s = τ_t/τ_s
Sc = Schmidt
T = temperature of heavy particles A and A^+ $[T = T^{\star}/(\gamma - 1)T^{\star}_{\text{ref}}]$
T_e = electron temperature, $T_e = T^{\star}_e/(\gamma - 1)T^{\star}_{\text{ref}}$
$\boldsymbol{U}$ = velocity, $\boldsymbol{U}^{\star}/U^{\star}_{\text{ref}}$
$U^{\star}_{\text{ref}}$ = sound velocity based on $T^{\star}_{\text{ref}}$
$\boldsymbol{V}^{\alpha}$ = diffusion velocity of species α
Y_α = mass fraction of species α, $Y^{\star}_\alpha/Y^{\star}_{\alpha,\text{ref}}$
γ = heat capacity ratio, 1.66 in argon
ϵ = dissipation rate of the fluctuating velocity field
λ = thermal conductivity of heavy particles
λ_e = thermal conductivity of electrons
μ, ν = dynamic viscosity and kinematic viscosity, $\mu = \mu^{\star}/\mu^{\star}_{\text{ref}}$
ρ = density, $\rho^{\star}/\rho^{\star}_{\text{ref}}$
τ_s = micromixing time of scalar s
τ_t = eddy breakup time
ψ = sample space variable
$\dot{\omega}$ = chemical rate

Subscripts

e = electrons
i or j = direction i or j
ref = reference
α = species α

Superscript

$\star$ = dimensional value

Introduction

MOST studies related to turbulence in plasma neglect the coupling between chemistry and turbulence. In these approaches the mean chemical source terms in the conservation equations for mean species concentrations are assumed to be functions of the averaged temperatures and species density. This is probably a too restrictive hypothesis as, for example, in thermal plasma jets.[1] The coupling between chemistry and turbulence in ionized gas could also be an important phenomenon in low-pressure plasma jets (arcjet) or in spatial-related applications, e.g., reentry of spatial vehicles or ballistic missiles. Studies have been devoted to this latter situation in the 1960s,[2,3] focusing on the electron density fluctuation, which is a key parameter for the prediction of radar signature. The previously proposed approaches are somewhat cumbersome and cannot be easily implemented in today's numerical solvers.

In the present work some simplifications to the complex problem of a turbulent plasma have been brought: 1) the test gas is argon, of which physical properties are well-known; 2) the level of ionization is low (below 1%); 3) no charge separation occurs; 4) no radiative effects are included; and 5) the electric field induced by turbulence is negligible.

When evaluating a mean source term in this turbulent plasma, in theory, one should account for fluctuations of chemical species and temperatures. In terms of modeling, a parallel can be drawn between this problem and turbulent combustion where the one-point probability density function (pdf) has proved to be an effective tool. Introducing pdfs, source terms defined at a point can be dealt with in a closed and exact form.[4–6] Nevertheless, all terms involving two-point information need to be modeled (diffusion of species, thermal conduction, radiative fluxes, etc.). Therefore with pdfs, the effects of fluctuations on chemical sources are directly included, whereas turbulent micromixing (small-scale diffusion) mechanisms need a closure. The extension of the pdf approach to turbulent ionized gases requires some fundamental studies to understand these unknowns.

To address this issue, we have used direct numerical simulation (DNS). DNS resolves all of the scales (time and length) of turbulence for moderate Reynolds numbers, and from the resulting data behavior of unknown terms can be studied.[7,8] A DNS solver previously developed for studying turbulence and turbulent combustion[9,10] has been adapted to treat the situation of a monoatomic plasma.

The paper is organized as follows. First, we present the governing equations for the plasma. In the subsequent section the pdf transport

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equation is derived, and a closure for turbulent micromixing is proposed. Finally, DNS databases are used to estimate the capability of the modeled pdf transport equation of capturing the main properties of the turbulent plasma.

Governing Equations

In this section we give the transport equations used to perform DNS of the turbulent plasma. From this set of equations, the corresponding pdf transport equation is also derived.

DNS

Simulations are three-dimensional and have been performed on a 65^3 grid, with periodic conditions on all boundaries of the domain. We use a fully compressible DNS code developed by Guichard et al.[9] The numerical method is third-order accurate in time and includes a sixth-order-accurate compact scheme for spatial discretization.[11] The equations[12] are detailed below in a nondimensional form:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho U_j}{\partial x_j} = 0 \tag{1}$$

$$\frac{\partial \rho U_i}{\partial t} + \frac{\partial \rho U_i U_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial \tau_{ij}}{\partial x_j} \tag{2}$$

$$\frac{\partial E}{\partial t} + \frac{\partial E U_j}{\partial x_j} = -\frac{\partial p U_j}{\partial x_j} + \frac{1}{Re}\frac{\partial}{\partial x_j}\left(\tau_{ij}U_i + \frac{\mu}{Pr}\frac{\partial T}{\partial x_j} + \frac{\mu}{Pr}\frac{\lambda_e^\star}{\lambda^\star}\frac{\partial T_e}{\partial x_j}\right) - \sum_{\alpha = A, A^+, e}\left[\frac{\partial}{\partial x_j}(\rho_\alpha V_j^\alpha h_\alpha)\right] - d_{\text{ion}}\dot{\omega}_{A^+}e^0_{A^+} \tag{3}$$

$$\frac{\partial \rho Y_{A^+}}{\partial t} + \frac{\partial \rho Y_{A^+} U_j}{\partial x_j} = \dot{\omega}_{A^+} - \frac{\partial}{\partial x_j}(\rho_{A^+} V_j^{A^+}) \tag{4}$$

$$\frac{\partial \rho_e e_e}{\partial t} + \frac{\partial \rho_e e_e U_j}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\mu}{d_{\text{ion}} RePr}\frac{\lambda_e^\star}{\lambda^\star}\frac{\partial T_e}{\partial x_j}\right) - \frac{\partial}{\partial x_j}(p_e U_j) - en_e(U_j + V_j^e)E_j^{el} - \frac{\partial}{\partial x_j}(\rho_e h_e V_j^e) + e_e\dot{\omega}_{A^+} - e_{\text{rec}}\dot{\omega}_{A^+} + Q_{\text{exc}} \tag{5}$$

where the pressure is given by

$$p = \left(\frac{\gamma - 1}{\gamma}\right)(\rho T + d_{\text{ion}}\rho_{A^+} T_e)$$

and the electric field (E^{el}) work is written as

$$-en_e(U_j + V_j^e)E_j^{el} = (U_j + V_j^e)\frac{\partial p_e}{\partial x_j}$$

The physical properties of the gas correspond to an argon plasma. The chemical source term accounts only for recombination of ions,[13] and it is suitable for electron temperatures below 5000 K:

$$\dot{\omega}_{A^+} = -Da\frac{(\rho Y_{A^+})^3}{[(\gamma - 1)T_e]^{4.5}} \tag{6}$$

The energy release in the electron gas related to ion recombination is only a fraction of the ionization energy ($e^{0\star}_{A^+} = 15.7$ eV) (Ref. 14):

$$e_{\text{rec}} \sim 0.165 e^0_{A^+}$$

and, with a temperature of reference equal to 1000 K,

$$e^0_{A^+} = e^{0\star}_{A^+}/a^{2\star}_{\text{ref}} \sim 106$$

The term for exchange of thermal energy between modes is given by[12]

$$Q_{\text{exc}} = Da_{\text{relax}} d_{\text{ion}} \rho_e^2 (T - T_e)\sqrt{T_e}$$

where the contribution of collisions between neutral particles and electrons has been dropped. This assumption is not always valid, especially when the ionization degree is weak (below 0.1%).

The diffusion velocities of ions and electrons are equal and include ambipolar diffusion.[15] We may write, neglecting pressure and temperature gradients,

$$V_j^{A^+} = V_j^e = -\frac{\mu}{ReSc}\left(1 + \frac{T_e}{T}\right)\frac{1}{\rho Y_{A^+}}\frac{\partial Y_{A^+}}{\partial x_j}$$

Neutral particles diffusion is described by a Fick's law and the viscosity depends on temperature through the Sutherland law:

$$\mu^\star = 1.97 \times 10^{-6} T^{\star 0.5} SI \Rightarrow \mu = [(\gamma - 1)T]^{0.5} \tag{7}$$

Obtaining a correct estimation of the thermal conductivity of electrons valid over a wide range of ionization is a difficult problem in itself. We have retained the expression proposed by Spitzer and Harm.[16] This expression is acceptable for an ionization degree over 10^{-3} and is given as

$$\lambda_e^\star = 1.84 \times 10^{-10} T_e^{\star 2.5}\frac{1}{\ell n(1.25 \times 10^7 \sqrt{T_e^{\star 3}/N_e^\star})} SI \tag{8}$$

Typically, the logarithmic part of the expression is close to 7; then assuming a temperature of reference of 1000 K and $Pr = 0.66$, it follows that

$$\mu\lambda_e^\star/\lambda^\star = 0.05[(\gamma - 1)T_e]^{2.5} \tag{9}$$

Statistical Description of the Turbulent Plasma (pdf)

From the preceding equations the transport equation for the joint pdf of the thermochemical variables (Y_{A^+}, ei, $Y_e e_e$) can be easily derived following the methodology of Dopazo.[6] In the situation of homogeneous turbulence, one may write

$$\begin{aligned}\frac{\partial \bar{\rho}\langle P\rangle}{\partial t} &= \frac{\partial}{\partial \psi_1}\left\{\overline{\left[\frac{\partial}{\partial x_j}(\rho_{A^+} V_j^{A^+})|\underline{\phi} = \underline{\psi}\right]}\bar{P}\right\} - \frac{\partial}{\partial \psi_1}[\dot{\omega}_{A^+}(\underline{\psi})\bar{P}] - \frac{\partial}{\partial \psi_2}\left\{\overline{\left[\frac{\partial}{\partial x_j}\left(\frac{\mu}{RePr}\frac{\partial T}{\partial x_j}\right)|\underline{\phi} = \underline{\psi}\right]}\bar{P}\right\} \\ &- \frac{\partial}{\partial \psi_2}\left\{\overline{\left[\frac{\partial}{\partial x_j}\left(\frac{\mu\lambda_e^\star}{RePr\lambda^\star}\frac{\partial T_e}{\partial x_j}\right)|\underline{\phi} = \underline{\psi}\right]}\bar{P}\right\} + \frac{\partial}{\partial \psi_2}\{[d_{\text{ion}}\dot{\omega}_{A^+}(\underline{\psi})e^0_{A^+}]\bar{P}\} \\ &- \frac{\partial}{\partial \psi_3}\left\{\overline{\left[\frac{\partial}{\partial x_j}\left(\frac{1}{d_{\text{ion}}}\frac{\mu\lambda_e^\star}{RePr\lambda^\star}\frac{\partial T_e}{\partial x_j}\right)|\underline{\phi} = \underline{\psi}\right]}\bar{P}\right\} + \frac{\partial}{\partial \psi_3}\left\{\overline{\left[\frac{p_e}{\rho_e Y_e}\sum_{j=1}^{3}\frac{\partial}{\partial x_j}(\rho_e V_{ej})|\underline{\phi} = \underline{\psi}\right]}\bar{P}\right\} \\ &- \frac{\partial}{\partial \psi_3}(\{\dot{\omega}_{A^+}(\underline{\psi})[e_e(\underline{\psi}) - e_{\text{rec}}] + Q_{\text{exc}}(\underline{\psi})\}\bar{P})\end{aligned} \tag{10}$$

where $\phi = \{\phi_1, \phi_2, \phi_3\}$ corresponds to $\{Y_{A^+}, ei, Y_e e_e\}$ and the related set of sample space variables is $\psi = \{\psi_1, \psi_2, \psi_3\}$. The quantity $(\overline{F \mid \kappa})$ is the mean value of expression F conditional on the satisfaction of the relation κ.

In Eq. (10) several terms have been neglected: convection is not included because the flow is homogeneous and transport of enthalpy by diffusion in the total energy equation and terms including pressure gradient/velocity and pressure/velocity divergence correlations are omitted. The contributions of the latter of these are supposed to be small in the flow configuration that we consider. Equation (10) involves unknown conditional mean values of diffusive and conductive budgets characteristic of turbulent micromixing.[17] To begin, we wish to investigate the possibility of using, for those terms, the linear estimation model of Dopazo,[6] also known as the interaction by

exchange with the mean value (IEM) model. For instance, using this closure, the diffusive term is

$$\overline{\left[\frac{\partial}{\partial x_j}\left(\frac{\rho\nu}{ReSc}\frac{\partial \phi_\alpha}{\partial x_j}\right)|\underline{\phi}=\underline{\psi}\right)\Bigg]} = \bar{\rho}\frac{\langle\phi_\alpha\rangle - \psi_\alpha}{\tau_\alpha} \tag{11}$$

The key point is then the determination of the characteristic micromixing time τ_α. DNS may be used as a guideline to construct closures for τ_α in the case of a turbulent plasma.

Results and Discussion

Validity of the IEM Model

When performing DNS, the initial distribution of the scalar Y_{A^+} corresponds to a segregated field featuring pockets of more or less ionized fluid (Fig. 1) and is characterized by $\langle Y_{A^+}\rangle = 0.5$ and $\sqrt{\langle Y''^2_{A^+}\rangle} = 0.3$. The temperatures (Fig. 2) T and T_e are initially set equal and linearly dependent on Y_{A^+} [$(\gamma - 1)\langle T\rangle = 1.28$ and $(\gamma - 1)\sqrt{\langle T''^2\rangle} = 0.22$].

The initial turbulent velocity field described by a Passot–Pouquet turbulence spectrum is characterized by a turbulent Reynolds number $Re_t = Reu'l_t$, where u' is the imposed rms of the velocity field and where the integral length scale is defined by $l_t = u'^3/\bar{\epsilon}$. In all of the simulations presented, the initial turbulent Reynolds number is about 160. All along these simulations, we insure that the Kolmogorov length scale [$l_K = (\langle\nu\rangle^3/\langle\epsilon\rangle)^{1/4}$] is twice the mesh step and that the ratio L/l_t is greater than 1 (L size of the computational domain in all directions). Then, the dissipation of the turbulent kinetic energy is well described, and the statistics extracted from the simulations are reliable.

The initial pressure field is uniform and set to unity. The acoustic Reynolds number is set to 1000, Prandlt and Schmidt are equal to 0.66, and the ionization degree of reference d_{ion} is equal to 0.01. From this initial condition the turbulence is allowed to decay. For all results presented the time is normalized by the initial eddy turnover time ($0.5\,l_t/\mu'$).

Other parameters are $Da = 0$ and $Da_{\text{relax}} = 0$ (nonreactive case A), $Da = 7$ and $Da_{\text{relax}} = 1.10^6$ (reactive case B), and $Da = 7$ and $Da_{\text{relax}} = 1.10^3$ (reactive case C). Case A describes a pure diffusivity and conductivity problem. Reactive cases B and C correspond to situations where ions are recombining leading to a rise in temperature levels. Because of the small ionization degree, the effect of recombination on the heavy particle temperature will be small. The influence on electron temperature of recombination will then depend on the intensity of the coupling between energy modes. Case B corresponds to a strong coupling ensuring no departure from thermal equilibrium; the electron temperature rise will then be small. In case C the coupling is weak, then the energy released by recombination in the electron gas will not be totally shared with heavy particles, leading to a significant electron temperature rise. This case is close to a low-pressure plasma where thermal nonequilibrium is frequently encountered. Case B can be seen as representative of an atmospheric thermal plasma where the assumption of perfect equilibrium between the two temperatures is reasonable.

When the IEM model is chosen for expressing the conditional means of diffusive and conductive terms appearing in Eq. (10), one can write

$$\overline{\left[\frac{\partial}{\partial x_j}\left(\frac{\rho\nu}{RePr}\frac{\partial T}{\partial x_j}\right)|\underline{\phi}=\underline{\psi}\right)\Bigg]} = \bar{\rho}\frac{\langle T\rangle - \psi_T}{\tau_T} \tag{12}$$

$$\overline{\left[\frac{\partial}{\partial x_j}\left(\frac{\rho\nu\lambda_e^\star}{RePr\lambda^\star}\frac{\partial T_e}{\partial x_j}\right)|\underline{\phi}=\underline{\psi}\right)\Bigg]} = \bar{\rho}d_{\text{ion}}\frac{\langle T_e\rangle - \psi_{T_e}}{\tau_{T_e}} \tag{13}$$

$$\overline{\left\{\frac{\partial}{\partial x_j}\left[\frac{\rho\nu}{ReSc}\left(1+\frac{T_e}{T}\right)\frac{\partial Y_{A^+}}{\partial x_j}\right]|\underline{\phi}=\underline{\psi}\right)\Bigg\}} = \bar{\rho}\frac{\langle Y_{A^+}\rangle - \psi_1}{\tau_{Y_{A^+}}} \tag{14}$$

where the scalar micromixing times are given by[6]

$$\tau_T = \langle T''^2\rangle \Big/ \left\langle \frac{\nu}{RePr}\frac{\partial T''}{\partial x_i}\frac{\partial T''}{\partial x_i}\right\rangle$$

$$\tau_{T_e} = \langle T_e''^2\rangle \Big/ \left\langle \frac{\nu\lambda_e^\star}{d_{\text{ion}}RePr\lambda^\star}\frac{\partial T_e''}{\partial x_i}\frac{\partial T_e''}{\partial x_i}\right\rangle$$

$$\tau_{Y_{A^+}} = \langle Y''^2_{A^+}\rangle \Big/ \left\langle \frac{\nu}{ReSc}\left(1+\frac{Te}{T}\right)\frac{\partial Y''_{A^+}}{\partial x_i}\frac{\partial Y''_{A^+}}{\partial x_i}\right\rangle \tag{15}$$

Figures 3–5 display, for diffusive and conductive terms, the comparison between IEM closure and the evolution extracted from DNS. The results are given for the reactive case B. The same level of agreement is found at other times in the simulation and also for cases A and C. Then, for these turbulent plasmas, IEM closure proves to be adequate if the micromixing time is well-predicted. In the next section the DNS database is now used to provide information on these micromixing times.

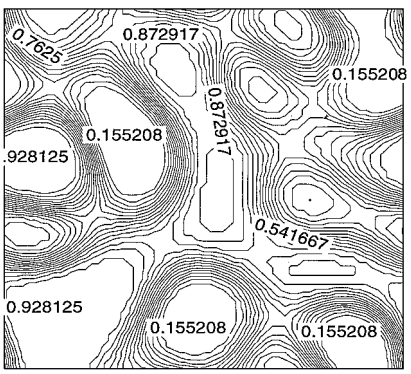

Fig. 1 Two-dimensional contour plot of the scalar Y_{A^+} at the initial time.

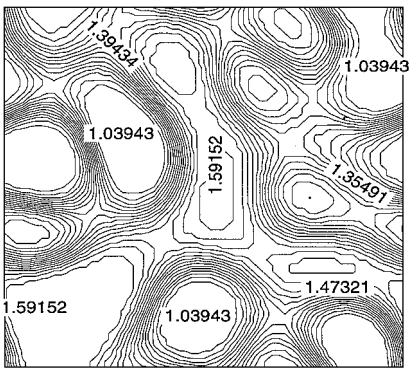

Fig. 2 Two-dimensional contour plot of the electron temperature $(\gamma - 1)T_e$ at the initial time.

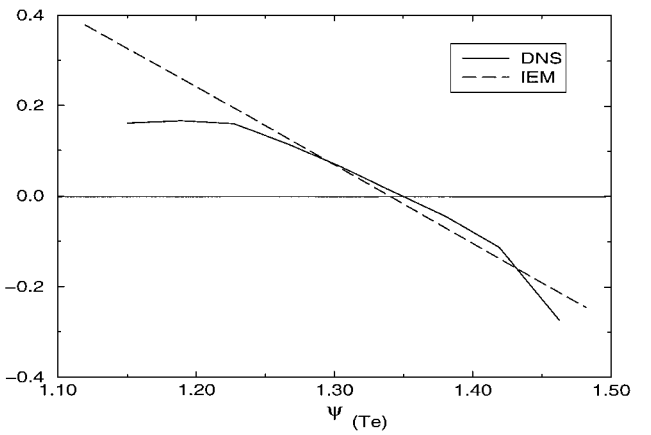

Fig. 3 $\{\overline{(\partial/\partial x_j)[(\rho\nu\lambda_e^\star/RePr\lambda^\star)(\partial T_e/\partial x_j)]|\underline{\phi}=\underline{\psi})}\}$, comparison between DNS and the IEM model (reactive case B, time = 1.6 eddy turnover times).

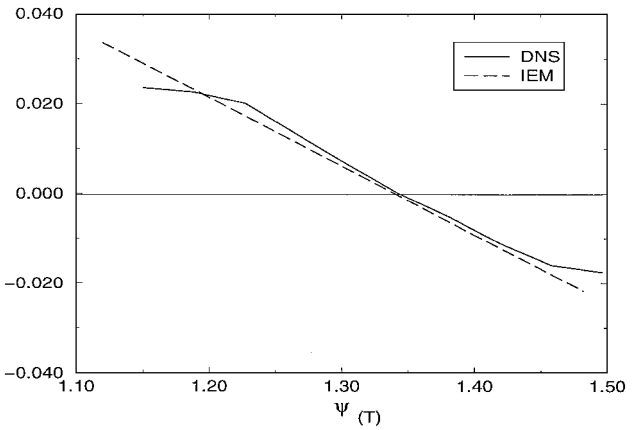

Fig. 4 $\{\overline{(\partial/\partial x_j)[(\rho\nu/RePr)(\partial T/\partial x_j)]|\underline{\phi}=\underline{\psi}}\}$, **comparison between DNS and the IEM model (reactive case B, time = 1.6 eddy turnover times).**

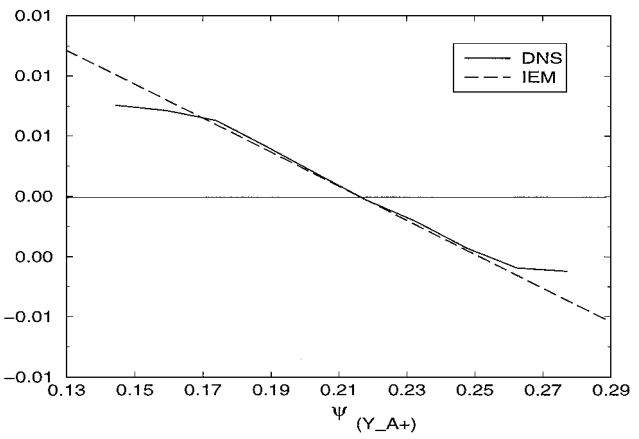

Fig. 5 $\{\overline{(\partial/\partial x_j)[(\rho\nu/ReSc)(1+T_e/T)(\partial Y_{A^+}/\partial x_j)]|\underline{\phi}=\underline{\psi}}\}$, **comparison between DNS and the IEM model (reactive case B, time = 1.6 eddy turnover times).**

Evolution of the Micromixing Times

A common assumption is to relate the micromixing time of the scalar to the eddy breakup time ($\tau_t = \langle k\rangle/\langle\epsilon\rangle$) through a proportionality relation:

$$r_\alpha = \tau_t/\tau_\alpha \sim C_\alpha \tag{16}$$

DNS studies (stationary isotropic turbulence with no mean gradient)[18] as well as experimental studies (spatially decaying grid turbulence)[19] have shown that after a transient regime C_α relaxes toward 1. These studies have been performed for Schmidt numbers equal to 0.7 (Ref. 18) and equal to 1 (Ref. 19). The transient behavior depends strongly on the initial velocity and scalar spectra. The modeling of the spectral relaxation of the scalar field is still an open problem that will not be addressed here (see, for example, Refs. 20 and 21).

In plasma, ambipolar diffusion tends to increase diffusion of ions, and we have roughly $Sc_{\text{amb}} = Sc_{A^+}/(1 + T_e/T) \sim 0.3$, which is the effective Schmidt number characterizing ionic diffusion, a value quite different of those used in the studies just mentioned.[18,19] The DNS databases generated with the conditions given next have been used to extract the temporal evolution of the mechanical-to-scalar time ratios (r_α).

As expected, the ratio corresponding to the micromixing of heavy particle temperature (r_T), which is characterized by $Pr = 0.66$, relaxes toward 1 (Fig. 6). The same trend is observed for ions (Fig. 6) despite the small ambipolar Schmidt number. These results are obtained from both the reactive and nonreactive simulations. Thus, even for quite low values of the Schmidt number, the assumption of a constant mechanical-to-scalar timescale ratio is acceptable after a transient regime. The simulation is stopped after 2.6 eddy turnover times because the integral length scale becomes too large

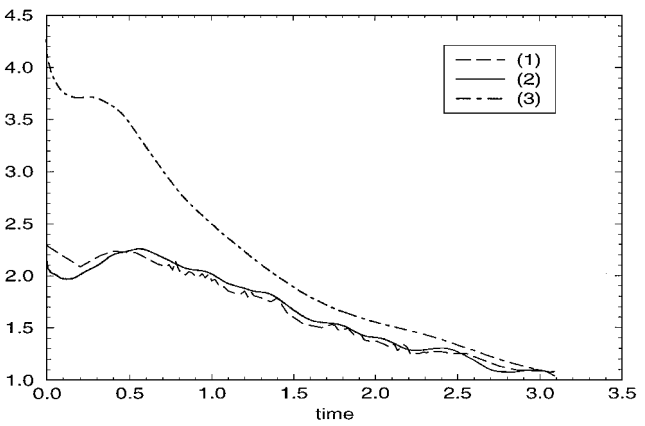

Fig. 6 Time evolution of the ratio of mechanical to scalar micromixing times (reactive case B): 1) $\tau_t/\tau^\star_{T_e}$, 2) τ_t/τ_T, and 3) $\tau_t/\tau_{Y_{A^+}}$.

compared to the size of the computational domain (at this time the ratio $\sqrt{\langle Y''^2_{A^+}\rangle}/\langle Y_{A^+}\rangle$ is equal to 0.08).

Turbulent micromixing for the electron temperature needs to be regarded with care. The first difficulty is that the evolution of viscosity depends on heavy temperature ($\sim T^{0.5}$), whereas electron conductivity depends on electron temperature ($\sim T_e^{2.5}$). The exponents are very different, and so, even if both temperatures are equal, the corresponding micromixing times are not related if the intensity of the fluctuations is large. The second difficulty is that the turbulent micromixing time is found to be strongly dependent on the source terms appearing in the electron energy balance equation [Eq. (5)]. Two extreme situations are distinguished: 1) if the energy exchange between heavy particles and electrons is low, then the small scales tend to be damped very quickly (Fig. 7) because of the important conductive term. In this situation, it is difficult to obtain satisfactory statistics because the integral length scale associated with electron temperature becomes of the same order as the size of the computational domain and 2) at the opposite limit, if the energy exchange is a dominant process, the small scales of the electron temperature are sustained by the heavy temperature ones (Fig. 8). Then, the statistical properties of both temperatures are identical. In this latter situation, combining Eqs. (7), (8), and (15) leads to the relation

$$\frac{C_1[(\gamma-1)\langle T_e\rangle]^{2.5}}{d_{\text{ion}}[(\gamma-1)\langle T\rangle]^{0.5}}\tau_{T_e} = \tau^\star_{T_e} \sim \tau_T \tag{17}$$

This simple relation is verified in Fig. 6 and should be applied when the intensity of fluctuations is not too high (below 10%) and when the mean temperatures ($\langle T\rangle$ and $\langle T_e\rangle$) are close together.

Pdf Modeling of a Turbulent Plasma

A Monte Carlo simulation[6] is used to solve the pdf transport equation [Eq. (10)]. The joint pdf is described with a set of 1000 stochastic particles of dimension 3 (Y_{A^+}, ei, $Y_e e_e$).

The initial pdf of T, T_e, and Y_{A^+} is the one used for generating the initial condition for DNS. Results are presented for reactive cases B and C. DNS and pdf method results are compared. The diffusive and conductive terms present in Eq. (10) are expressed with the IEM model using the turbulent micromixing times either issued from DNS [Eq. (15)] or calculated from the eddy breakup time [Eq. (16)] with $C_T = C_{Y_{A^+}} = 1$ and $C_{T_e} = 10$. This latter approximation allows for the inclusion of the strong conductivity of electrons.

Figures 9–13 display the evolution of mean values and rms of the ion mass fraction and of the temperatures. For all values and for both expressions of the micromixing turbulent times (directly issued from DNS or proportional to the eddy breakup time), the mean values are accurately described. The electron temperature for case B is not displayed because the strong coupling between thermal energy modes ensures a total thermal equilibrium. The mean electron temperature in case C (Fig. 12) shows at the beginning of the simulation a significant rise related to electron/ion recombination and then slowly relaxes toward lower values as electron energy is

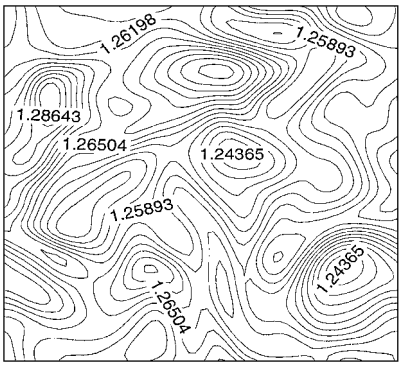

Fig. 7 Two-dimensional contour plot of the electron temperature $(\gamma - 1)T_e$ after one eddy turnover time (nonreactive case A).

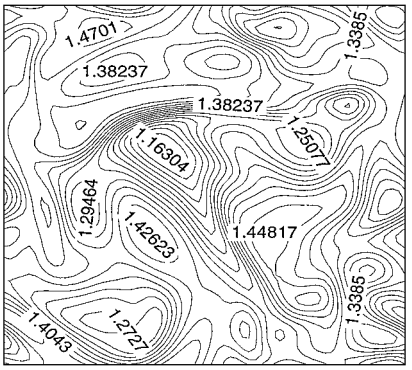

Fig. 8 Two-dimensional contour plot of the electron temperature $(\gamma - 1)T_e$ after 1.5 eddy turnover time (reactive case B).

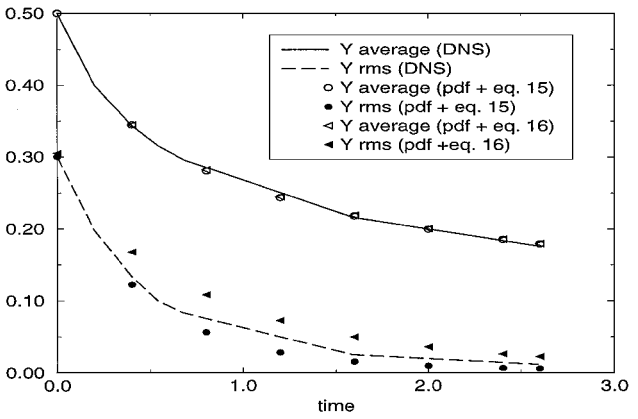

Fig. 9 Time evolution of $\langle Y_{A^+} \rangle$ and $\sqrt{\langle Y_{A^+}'^2 \rangle}$ (reactive case B).

given to the heavy particles through elastic collisions. We may also remark, comparing Figs. 9 and 11, that the rise in electron temperature observed in case C slows down the ion/electron recombination process.

The observation of the evolutions of the rms leads to more qualified statements. As expected, with an approximation [Eq. (16)] of the micromixing times, a less accurate prediction of the rms is achieved for the ion mass fraction (Figs. 9 and 11). However, the rms of the heavy species temperature (Figs. 10 and 13) is predicted with a better accuracy with the relation (16). In fact, to establish the pdf transport equation [Eq. (10)], the term $p(\partial u_j/\partial x_j)$ was neglected in the internal energy ei transport equation (3). This term extracted from DNS appears to be only slightly smaller than the conductive term (two or three times smaller) and to be of opposite sign. Then it slows the decay of temperature fluctuations. The relation (16) overestimates the micromixing time and then reproduces artificially the complex balance between conduction and the pressure/velocity divergence correlation.

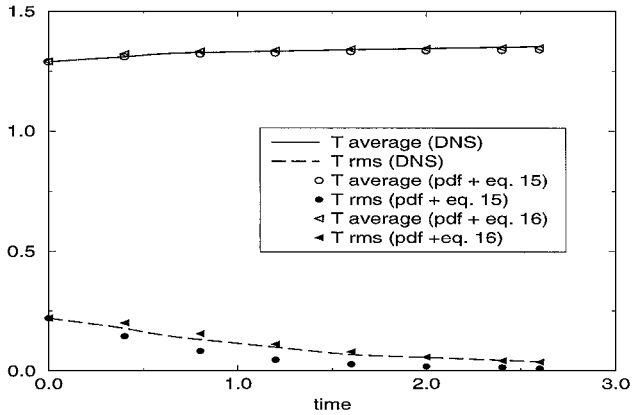

Fig. 10 Time evolution of $(\gamma - 1)\langle T \rangle$ and $(\gamma - 1)\sqrt{\langle T''^2 \rangle}$ (reactive case B).

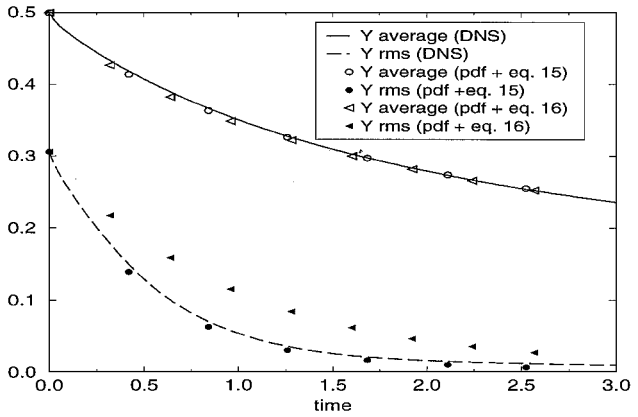

Fig. 11 Time evolution of $\langle Y_{A^+} \rangle$ and $\sqrt{\langle Y_{A^+}'^2 \rangle}$ (reactive case C).

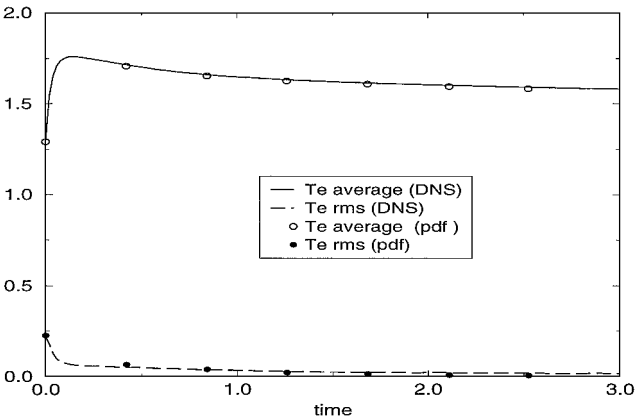

Fig. 12 Time evolution of $(\gamma - 1)\langle T_e \rangle$ and $(\gamma - 1)\sqrt{\langle T_e''^2 \rangle}$ (reactive case C).

The rms of the electron temperature in case C (Fig. 12) vanishes very quickly because of the strong conductivity of electrons, and the agreement between DNS and pdf method is satisfactory. Comparing the rms of the ion mass fraction (Figs. 9 and 11), we note that if the micromixing time extracted from DNS is used the agreement is better for case C than for case B, which implies that a good prediction of the rms of the controlling temperature in the chemical source term is also necessary.

The IEM model that has been used to close the pdf transport equation does not modify the shape of the pdf, which is initially a two-peaks pdf and evolves toward Gaussian shape in the DNS. Nevertheless, it appears to be sufficient to predict the mean value and the rms of the quantities qualifying the turbulent plasma, as soon as a good estimation of the turbulent micromixing time is provided.

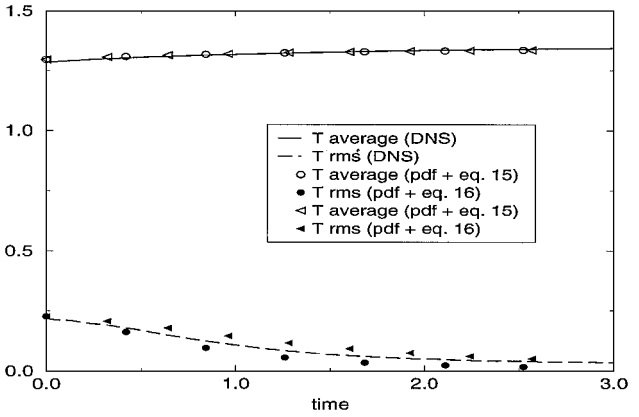

Fig. 13 Time evolution of $(\gamma - 1)\langle T \rangle$ and $(\gamma - 1)\sqrt{\langle T''^2 \rangle}$ (reactive case C).

Conclusion

The possibilities of applying the one-point pdf approach to the modeling of turbulent plasma have been investigated using DNS. The intricate coupling between the thermal energy exchange term and the conductive term appearing in the electron energy transport equation and the ion/electron recombination process has been shown. Encouraging results have been obtained when comparing DNS results with the solution of the pdf equation, where the diffusive and conductive terms are closed with the basic IEM model. It may then be concluded that using a pdf coupled with turbulence modeling of the flow may be an effective route to help in the design of practical systems involving turbulent nonequilibrium plasma flows.

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